

# Digital Logic Design

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## Refs :-

① Digital system : Principle of Applications  
by: Ronald J. Tocci

② Digital Design .  
by: M. Morris Mano

③ Introduction to Digital Computer Technology  
by: Louis

④ Any Books for Digital Design

# Number System

Decimal System :-

Digits : (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Radix (r) : (10)

- A decimal number such as 7392 represents a quantity equal to 7 thousands, plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds, etc, are powers of 10.

To be more exact 7392 should be written as:-

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

In general a number with a decimal point is represented by a series of coefficients as follows:-

$$a_5 a_4 a_3 a_2 a_1 . a_{-1} a_{-2} a_{-3}$$

The  $a_j$  coefficients are one of the ten digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and the subscript (j) gives the place value and, hence, the power of 10 by which the coefficient must be multiplied

$$10^5 a_5 + 10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10^1 a_1 + 10^0 a_0 + 10^{-1} a_{-1} + 10^{-2} a_{-2} + 10^{-3} a_{-3}$$

Ex)

$$\begin{array}{c} (462)_{10} \\ \swarrow \quad \downarrow \quad \searrow \\ 10^2 \quad 10^1 \quad 10^0 \end{array}$$

$$\Rightarrow 4 \times 100 + 6 \times 10 + 2 \times 1 = (462)_{10}$$

In general :-

$$N = a_{q-1} r^{q-1} + a_{q-2} r^{q-2} + \dots + a_1 r^1 + a_0 r^0$$

This eq. to find the value of each Number

- For Decimal

$$N_{10} = \sum_{i=-\infty}^{\infty} k_i 10^i$$

- Binary System

Digits = (0, 1)

$r = 2$

$$N_2 = \sum_{i=-\infty}^{\infty} k_i 2^i$$

In Binary number each coefficient  $a_j$  is multiplied by  $2^j$  for example, the decimal equivalent of the Binary Number (11010.11) is (26.75)

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} \\ = (26.75)_{10}$$

In general, a number expressed in base- $r$  system has coefficients multiplied by power of  $r$ :

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \dots + a_2 \cdot r^2 + a_1 \cdot r^1 + a_0 \cdot r^0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \dots + a_{-m} \cdot r^{-m}$$

The Coefficient  $a_j$  range in Value from 0 to  $r-1$

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100

Ex) Convert (13-30) to the Binary system? H.W

Ex)

$$\begin{array}{c} (101)_2 \\ \swarrow \quad \downarrow \quad \searrow \\ 2^2 \quad 2^1 \quad 2^0 \\ \circ \quad \circ \quad \circ \end{array} \Rightarrow 1 \times 4 + 0 \times 2 + 1 \times 1 = (5)_{10}$$

$$\circ \quad (101)_2 \longrightarrow (5)_{10}$$

Ex)  $(101010111)_2 \longrightarrow (x)_{10}$

$$1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$256 + 0 + 64 + 0 + 16 + 0 + 4 + 2 + 1 = (343)_{10}$$

$\therefore (101010111)_2 \longrightarrow (343)_{10}$

Ternary System :-

Digit: (0, 1, 2)

$$r = 3$$

$$N_3 = \sum_{i=-\infty}^{\infty} k_i 3^i$$

Ex)  $(102)_3 \longrightarrow (x)_{10}$

$$1 \times 3^2 + 0 \times 3^1 + 2 \times 3^0$$

$$9 + 0 + 2 = (11)_{10}$$

$\therefore (102)_3 \longrightarrow (11)_{10}$

- An example of base-5 Number is

$$(4021)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0$$

$$= (511)_{10}$$

Note that the coefficient values for base 5 can be only

Digit = (0, 1, 2, 3, 4)

Radix = (r = 5)

## Octal system

Digits (0, 1, 2, 3, 4, 5, 6, 7)

$$r = 8$$

$$N_8 = \sum_{i=-\infty}^{\infty} k_i 8^i$$

Ex)  $(167)_8$

$$\begin{array}{c} (167)_8 \\ \swarrow \quad \searrow \\ 8^2 \quad 8^1 \quad 8^0 \end{array}$$

$$1 \times 64 + 6 \times 8 + 7 = (119)_{10}$$

## Hexa Decimal system

Digits: (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

$$r = 16$$

Ex)  $(29)_{16} \longrightarrow (x)_{10}$

$$\begin{array}{c} (29)_{16} \\ \swarrow \quad \searrow \\ 16^1 \quad 16^0 \end{array}$$

$$2 \times 16 + 9 \times 1 = (11)_{10}$$

Ex)  $(4CBF)_{16} \longrightarrow (x)_{10}$

$$\begin{array}{c} (4CBF)_{16} \\ \swarrow \quad \downarrow \quad \downarrow \quad \searrow \\ 16^3 \quad 16^2 \quad 16^1 \quad 16^0 \end{array}$$

$$4 \times 16^3 + 12 \times 16^2 + 11 \times 16^1 + 15 \times 1 =$$

$$16384 + 3072 + 176 + 15 = (19551)_{10}$$



Counting in Number System:-

Ex) write the first 23 decimal digits in base-5.

Digit = (0, 1, 2, 3, 4)

= (0, 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42).

Ex) write the first 28 decimal digits in base-9.

Digit = (0, 1, 2, 3, 4, 5, 6, 7, 8)

= (0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30).

Ex) write from 7 to 44 decimal digits in base-16.

Digits = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

= (7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C).

Ex) write first 15 number in base-12.

Digits = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B).

= (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, 10, 11, 12).

Numbers with different bases: -

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

## Number Base Conversions

Conversions from any number system to decimal system.

To do that by — Conversion the Integer Number.

— : : Fraction : .

Ex) Convert the following number  $(2121)_3$  to decimal?

$$(2121)_3 \longrightarrow (X)_{10}$$

— Integer part

$$\begin{aligned} (X)_{10} &= 1 \times 3^0 + 2 \times 3^1 + 1 \times 3^2 + 2 \times 3^3 \\ &= 1 + 6 + 9 + 54 = (70)_{10} \end{aligned}$$

Ex) Convert the number  $(0.342)_5$  to decimal system?

$$(0.342)_5 \longrightarrow (X)_{10} \quad (\text{Fraction No.})$$

$$\begin{aligned} (X)_{10} &= 3 \times 5^{-1} + 4 \times 5^{-2} + 2 \times 5^{-3} \\ &= 0.6 + 0.16 + 0.016 = (0.776)_{10} \end{aligned}$$

Ex) Convert the number  $(0.10101)_2 \longrightarrow (X)_{10}$

$$\begin{aligned} (X)_{10} &\rightarrow 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} + 1 \times 2^{-5} \\ &= 0.5 + 0 + 0.125 + 0 + 0.03125 \\ &= (0.656)_{10} \end{aligned}$$

Ex) Convert the number  $(0.ABC2)_{14} \longrightarrow (X)_{10}$

$$\begin{aligned} (X)_{10} &= A \times 14^{-1} + B \times 14^{-2} + C \times 14^{-3} + 2 \times 14^{-4} \\ &= 0.714 + 0.056 + 0.00437 + 0.000052 \\ &= (0.774)_{10} \end{aligned}$$

Ex) Convert the following Number from  $(4576)_8$  to decimal system?

$$\begin{aligned}(X)_{10} &= 4 \times 8^3 + 5 \times 8^2 + 7 \times 8^1 + 6 \times 8^0 \\ &= 2048 + 320 + 56 + 6 \\ &= (2430)_{10}\end{aligned}$$

Ex) Convert the following Number from  $(2ABCDEF)_{16}$  to decimal system?

$$\begin{aligned}(2ABCDEF) &\rightarrow (X)_{10} \\ (X)_{10} &= 2 \times 16^4 + A \times 16^3 + B \times 16^2 + C \times 16^1 + F \times 16^0 \\ &= 131072 + 40960 + 2816 + 192 + 15 \\ &= (175055)_{10}\end{aligned}$$

To convert any Number from the decimal system to any system we divided the Number into two parts:-

① The Integer Part.

Divide the Number by the base of system and take the Remainder, and read the Number from bottom to top.

② The Fraction Part.

Doing that by Multiply by the base of system and take the Integer part after each Multiplication and Multiply the remainder of fraction by the base until three digits.

Convert the Number  $(3486.248)_{10}$  to Binary system?

Integer + Fraction		
/	\	
3486	0.248	
		Remainder
2	3486	0
2	1743	1
2	871	1
2	435	1
2	217	1
2	108	0
2	54	0
2	27	1
2	13	1
2	6	0
2	3	1
2	1	1
	0	

Read from here

$$(3486)_{10} \rightarrow (110110011110)_2$$

To insure :-

$$(110110011110)_2 \rightarrow (x)_{10}$$

$$(x)_{10} = 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 + 0 \times 2^5 + 0 \times 2^6 + 1 \times 2^7 + 1 \times 2^8 + 0 \times 2^9 + 1 \times 2^{10} + 1 \times 2^{11} = (3486)_{10}$$

## - Fraction Part

$$0.248 \times 2 = 0.496 \quad 0$$

$$0.496 \times 2 = 0.992 \quad 0$$

$$0.992 \times 2 = 1.984 \quad 1$$

$$\therefore (0.248)_{10} \rightarrow (0.001)_2$$

$$\therefore (3486.248)_{10} \rightarrow (11011001111.001)_2$$

Number base conversions: -

- Conversion from  $N$  system to decimal.

- Conversion from decimal system to  $N$  system.

Ex) Convert the same number  $(3486.248)_{10}$  to base 5 and base 8?

Sol<sup>n</sup>)

$$(3486.248)_{10} \rightarrow (x)_5$$

5	3486	$\frac{R}{0}$	
5	697	1	↑
5	139	2	
5	27	4	
5	5	2	
5	1	0	
5	0	1	

$$(0.248)_{10} \rightarrow (x)_5$$

$$(3486)_{10} \rightarrow (102421)_5$$

$$0.248 \times 5 = 1.24$$

$$0.248 \times 5 = 1.2$$

$$0.2 \times 5 = 1.0$$

$$0.0 \times 5 = 0$$

$$\therefore (3486.248)_{10}$$

$$\longrightarrow (X)_5$$

$$\longrightarrow (102421.244)_5$$

$$(3486.248)_{10} \longrightarrow (X)_8 \quad \text{H.W}$$

Ex) Convert the following number  $(5213.879)_{10}$  to Hexadecimal system? H.W

## Octal and Hexadecimal Numbers

\* Conversion from Binary to Octal system

To convert Binary number to Octal, the Binary No. will be divide to groups each group from three digit as in table :-

No. in Octal	Group in Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

## \* Conversion from Octal to Binary System.

Ex) Convert these Numbers:-

$$a) (011101011110)_2 \rightarrow (x)_8$$

$$b) (2456)_8 \rightarrow (x)_2$$

$$a) \left( \frac{011}{3} \quad \frac{101}{5} \quad \frac{011}{3} \quad \frac{110}{6} \right)_2$$

$$\therefore (011101011110)_2 \rightarrow (3536)_8$$

$$b) (2456)_8 \rightarrow (x)_2$$

$$\begin{array}{cccc} & (2) & (4) & (5) & (6) \\ & / & / & | & \backslash \\ 010 & 100 & 101 & 110 & \end{array} \rightarrow (010100101110)_2$$

$$c) (000111011100)_2 \rightarrow (x)_8 \quad \text{H.w}$$

$$d) (24573)_8 \rightarrow (x)_2 \quad \text{H.w}$$

## \* Conversion from Binary to Hexadecimal System.

When we convert numbers in Binary to Hexadecimal, we divided the number in Binary System into groups, each group consist from four digit (Binary digit) as shown in next table:-



No. in Hexadecimal	Group in Binary
0	0000
1	0001
2	0010
...	...
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Ex) Convert these Numbers or Do these Operations?

a)  $(011101111011101)_2 \longrightarrow (x)_{16}$

b)  $(101010100001101)_2 \longrightarrow (x)_{16}$  H.w

c)  $(ABC8976)_{16} \longrightarrow (x)_2$

d)  $(345AFFE8)_{16} \longrightarrow (x)_2$  H.w

Sol<sup>n</sup>: a)  $(\underbrace{0111}_7 \underbrace{0111}_7 \underbrace{1011}_B \underbrace{1101}_D) \longrightarrow (77BD)_{16}$

c)  $(ABC8976)_{16} \longrightarrow (101010111001000100101101)_2$   
 $(\underbrace{1010}_{A} \underbrace{1011}_{B} \underbrace{1100}_{C} \underbrace{1000}_{8} \underbrace{1001}_{9} \underbrace{0111}_{7} \underbrace{0110}_{6})_2$

Note:-

Conversion from special system to special system (Not 2, 8, 16) as Below :-

$$E+) \quad (a) \quad (1243)_5 \longrightarrow (x)_9$$

$$(1243)_5 \longrightarrow (x)_{10} \longrightarrow (x)_9$$

$$(x)_{10} = 3 \times 5^0 + 4 \times 5^1 + 2 \times 5^2 + 1 \times 5^3$$

$$= 3 + 20 + 50 + 125 = 198$$

$$\therefore (198)_{10} \longrightarrow (x)_9$$

9	198	$\frac{R}{0}$	↑
9	22	4	
9	2	2	
	0		

$$(198)_{10} \longrightarrow (240)_9$$

$$\therefore (1243)_5 \longrightarrow (198)_{10} \longrightarrow (240)_9$$

$$E+) \quad (4567)_9 \longrightarrow (x)_{16} \quad \text{H.W}$$

$$E+) \quad (BCE2.BBA)_{13} \longrightarrow (x)_7 \quad \text{H.W}$$

$$E+) \quad (ABCEE.FFF)_{16} \longrightarrow (x)_4 \quad \text{H.W}$$

Complements :-

Complements are used in digital computers for simplifying the subtraction operation and for logical manipulations.

There are two types of complements for each base- $r$  system.

① The  $r$ 's complement.

② The  $(r-1)$ 's complement.

① The  $r$ 's complement

Given a positive number  $N$  in base  $r$  with an integer part of  $n$  digits, the  $r$ 's complement of  $N$  is defined as

$$r^n - N \quad \text{for } N \neq 0 \quad \& \quad 0 \text{ for } N = 0$$

- The  $10$ 's complement of  $(52520)_{10}$  is

$$10^5 - 52520 = 4780$$

The number of digit in the number is  $n = 5$

- The  $10$ 's complement of  $(0.3267)_{10}$  is

$$1 - 0.3267 = 0.6733$$

No integer part, so  $10^n = 10^0 = 1$

- The  $10$ 's complement of  $(25.639)_{10}$  is

$$10^2 - 25.639 = 74.361$$

- The  $2$ 's complement of  $(101100)_2$  is

$$(2^6)_{10} - (101100)_2 = (1000000 - 101100)$$

$$= 010100$$

- The  $2$ 's complement of  $(0.0110)_2$  is

$$(1 - 0.0110)_2 = 0.1010$$

The  $(r-1)$ 's Complement.

Given a positive number  $N$  in base  $r$  with an integer part of  $n$  digits and a fraction part of  $m$  digits, the  $(r-1)$ 's complement of  $N$  is defined as

$$r^n - r^{-m} - N$$

- The 9's complement of  $(52520)_{10}$  is

$$(10^5 - 1 - 52520) = 99999 - 52520 = 47479$$

No fraction part, so  $10^{-m} = 10^0 = 1$

- The 9's complement of  $(0.3267)_{10}$  is

$$(1 - 10^{-4} - 0.3267) = 0.9999 - 0.3267 = 0.6732$$

No integer part, so  $10^n = 10^0 = 1$

- The 9's complement of  $(25.639)_{10}$  is

$$(10^2 - 10^{-3} - 25.639) = 99.999 - 25.639 = 74.360$$

- The 1's complement of  $(101100)_2$  is

$$(2^6 - 1)_{10} - (101100)_2 = (111111 - 101100)_2 = 010011$$

- The 1's complement of  $(0.0110)_2$  is

$$(1 - 2^{-4})_{10} - (0.0110)_2 = 0.1111 - 0.0110 = 0.1001$$

Note:-

9's Complement } used for decimal  
10's Complement }

1's Complement } used for Binary  
2's Complement }

Notes:-

- R-System Complement

Here we consider the base  $R$  number system to find the complement as follow:-

① We deal with just the numbers that have negative sign, the numbers that have positive sign leave it without any operations.

② Subtract the negative number from the higher number that this system is contain  $(R-1)$ , then add  $(1)$  to the result from the right side.

③ After addition with the second number, if the result of this operation has carry, this carry will neglegable.

Ex) Do the operation by using  $R$  complement.

$$\begin{array}{r} + (43567)_8 \\ - (23456)_8 \end{array}$$

$$\begin{array}{r} 77777 \\ - 23456 \\ \hline 54321 \\ \quad 1+ \\ \hline 54322 \end{array}$$

43567  
54322

20111

Carry  
1  
↓  
Cancel  
Ex

Do the operation by using 2's complement

1111101  
- 1100110

1100110 1's comp.  
0011001  
1 +

0011010

1111101  
+ 0011010

Carry  
1  
↓  
Cancel

00110111

- Complement of Binary system

(a) Convert all 0's → 1's and add the sum with 1  
or 1's → 0's

(b)

1100110  
↓ ↓ ↓ ↓ ↓ ↓  
0011010 2's comp.

R-1 System:-

Here we consider the  $(r-1)$  to find the complement as

- ① We deal with Just the numbers that have the negative sign, the numbers that have positive sign it is leave without any operations.
- ② Subtract the negative number from the higher number that this system is contain  $(r-1)$ .
- ③ After addition with the second number, if the result of this operation has Carry, the Carry will add to the result number from the Right side of this No.

Ex) Do the operation below by using R-1 comp.

$$\begin{array}{r}
 (ABCEF)_{16} \\
 - (48F9D)_{16} \\
 \hline
 FFFFF \\
 - 48F9D \\
 \hline
 B7062
 \end{array}$$

$$\begin{array}{r}
 \text{Carry} \\
 \boxed{1} \\
 \downarrow \\
 \begin{array}{r}
 ABC EF \\
 B7062 \\
 \hline
 62D51 \\
 \downarrow \\
 \hline
 \downarrow 62D52
 \end{array}
 \end{array}$$

Ex) Do the operation by using 1's complement.

$$\begin{array}{r}
 1110110 \\
 - 1001100 \\
 \hline
 1001100 \\
 0110011 \downarrow \text{comp.} \\
 \hline
 1110110 \\
 + 0110011 \\
 \hline
 \text{Carry } \boxed{1} \\
 0101001 \\
 \hline
 \downarrow \\
 0101010
 \end{array}$$

Codes :-

Codes have been used for security reasons, so that others will not be able to read the message.

There are several types of codes, such as:-

BCD (Binary Codes Decimal) (8421)

it used the binary number system to specified the decimal no. (0-9), 4-binary bits are require



Decimal	BCD (4-bit)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

- The (8421) is a type of binary codes decimal (BCD) and is composed of four bit representing the decimal number (0-9).

The weight of the four bits is  $[2^3 \ 2^2 \ 2^1 \ 2^0]$   
 $\downarrow$  MSB  $\downarrow$  LSB

The main advantage of this codes is the easy of conversion to and from decimal.

Ex) Find the decimal no. represented the following

BCD

BCD Coded	Decimal No.
0011 0110	36
0111 1000	78

## Complementary Codes:-

There are several complementary codes such as (2421) is a common complementary code, it is similar to (8421) code, in the sense that the position of the digits carry a non-weight except that the (MSB) carries a weight of (2) instead of (8). The advantage is that it is complementing which is used in Arithmetical operations.

The (7421) code is another code in which the weight of the (MSB) is (7) which has useful property that it has a minimum no. of (1's) which could be useful for economy in power consumption.

Ex) represent the following no. in 2421

$$\begin{array}{r} 379 \\ \hline 10 \end{array}$$

$$(0011 \quad 1101 \quad 1111)$$

2421

## EX-3 Code (EXcess-3 Code)

The EX-3 code is a digital code that is derived by adding (3) to each decimal digit and then converting the result to (4) bit binary.

EX-3 is an unweighting code. The table below show that:-

Decimal No.	Ex-3 Code
0	0 0 1 1
1	0 1 0 0
2	0 1 0 1
3	0 1 1 0
4	0 1 1 1
5	1 0 0 0
6	1 0 0 1
7	1 0 1 0
8	1 0 1 1
9	1 1 0 0

Ex) Represent the following No. in Ex-3 code?

Decimal No.	Ex-3 code
5	1 0 0 0
27	0 1 0 1    1 0 1 0
310	0 1 1 0    0 1 0 0    0 0 1 1

Gray Code :-

(un weighted code)

Gray Code an un weighted code, which means that there is no specific weights, assigned to bit position. The Gray Code have only a single bit change from





In Ex-3 Code each code ch/ls in Ex-3 is three larger than BCD

Decimal	Ex-3
0	0011
1	0100
⋮	⋮
9	1100
22	0101 0101
86	1011 1001

### Sign Magnitude Numbers:-

which is (bit) for sign to the number, if the No. is (negative) the sign is (1), if the No. is (positive) the sign is (0), to find the complements (r's) or (r-1)'s for each number.

Ex) obtain the sign magnitude, Signed 1's comply 2's compl. representation of the number -38?

sign magnitude	1	00110	
	1	011001	1's comp.
	1	011001	2's comp.
		1 <sup>+</sup>	
	1	011010	

- Ex) (a)  $\frac{5}{1}$  0110      Sol<sup>n</sup>      (a) -6
- (b) 0 111.11      (b) +7.75

Ex) Use 2's comp. to perform  $75 - 35$

1's comp

S.b	64	32	16	8	4	2	1
1	0	1	0	0	0	1	1
0	1	0	0	1	0	1	1
1	1	0	1	1	1	0	0
0	1	0	0	1	0	1	1
1	1	0	1	1	1	0	0
							1

2's comp.

1	1	0	1	1	1	0	1
0	1	0	0	1	0	1	1
0	0	1	0	1	0	0	0

Carry  
Cancel 1

∴ Sol<sup>n</sup> is (40) and (S.B) is (+).

Ex)  $-6 - 7$

S.b	addition bit	4	2	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	1
1	1	0	0	0
1	0	0	0	1
				1
				0

← negative ← 1

1  $-_{13}$  1101

# Arithmetic Operations:-

## \* Binary Operations:-

### (a) Addition

+	0	1
0	0	1
1	1	0+c

Ex)  $(37)_{10} + (13)_{10}$

1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	1	0

### (b) Subtraction

-	0	1
0	0	1
1	1+b	0

Ex)  $(22)_{10} - (12)_{10}$

1	0	1	1	0
0	1	1	0	0
0	1	0	1	0

### (c) Multiplication

1	0	1	1	0	1	1
				1	0	1
1	0	1	1	0	1	1
1	0	0	0	0	0	0
1	0	1	1	0	1	1
1	1	1	0	0	0	1



ⓓ Division

$$\begin{array}{r}
 \phantom{101} \overline{) 001001} \\
 \underline{101} \phantom{00} \\
 \phantom{101} 000 \phantom{00} \\
 \phantom{101} \phantom{00} \underline{101} \\
 \phantom{101} \phantom{00} \phantom{101} \phantom{00} \\
 \phantom{101} \phantom{00} \phantom{101} \phantom{00} \underline{000}
 \end{array}$$

Addition and Subtraction of BCD Codes:-

Ex) Represent 9's complement of 2421 code (271)<sub>10</sub>

$$(271)_{10} = (728)_{9's \text{ comp.}} = 1101 \ 0010 \ 1110$$

ⓓ BCD Addition:-

Ex) 36 + 41

$$\begin{array}{r}
 \phantom{00} 0011 \phantom{00} \phantom{00} \\
 \phantom{00} 0100 \phantom{00} \phantom{00} \\
 \hline
 \phantom{00} 0111 \phantom{00} \phantom{00} \\
 \phantom{00} \phantom{00} 7 \phantom{00} \phantom{00}
 \end{array}
 \qquad
 \begin{array}{r}
 \phantom{00} 0110 \phantom{00} \\
 \phantom{00} 0001 \phantom{00} \\
 \hline
 \phantom{00} 0111 \phantom{00} \\
 \phantom{00} \phantom{00} 7 \phantom{00}
 \end{array}$$

Ex) 36 + 25

$$\begin{array}{r}
 \phantom{00} 0010 \phantom{00} \phantom{00} \\
 \phantom{00} 0011 \phantom{00} \phantom{00} \\
 \hline
 \phantom{00} 0101 \phantom{00} \phantom{00} \\
 \phantom{00} \phantom{00} 7 \phantom{00} \phantom{00}
 \end{array}
 \qquad
 \begin{array}{r}
 \phantom{00} 0101 \phantom{00} \\
 \phantom{00} 0110 \phantom{00} \\
 \hline
 \phantom{00} 1011 \phantom{00} \\
 \phantom{00} 0110 > 9 \\
 \hline
 \phantom{00} 0001 \phantom{00} \\
 \phantom{00} \phantom{00} 1 \phantom{00} \phantom{00}
 \end{array}
 \qquad
 = (61)$$